

SAMPLE PAPER 7

Leaving Certificate

Mathematics

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1

(25 marks)

- (a) (i)** Write out the first 10 natural numbers that are perfect squares.

[illegible]

- (ii)** Show, in general, that the difference between any two consecutive square natural numbers is always an odd number.

[illegible]

- (b)** Show that:

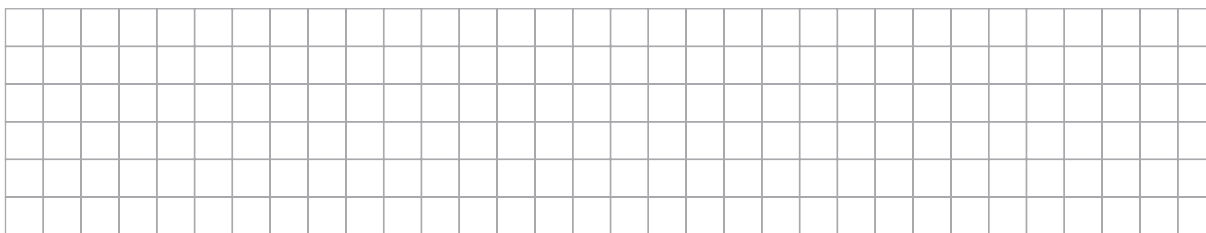
(i) $1^2 + 2^2 = \frac{2 \times 3 \times 5}{6},$

[illegible]

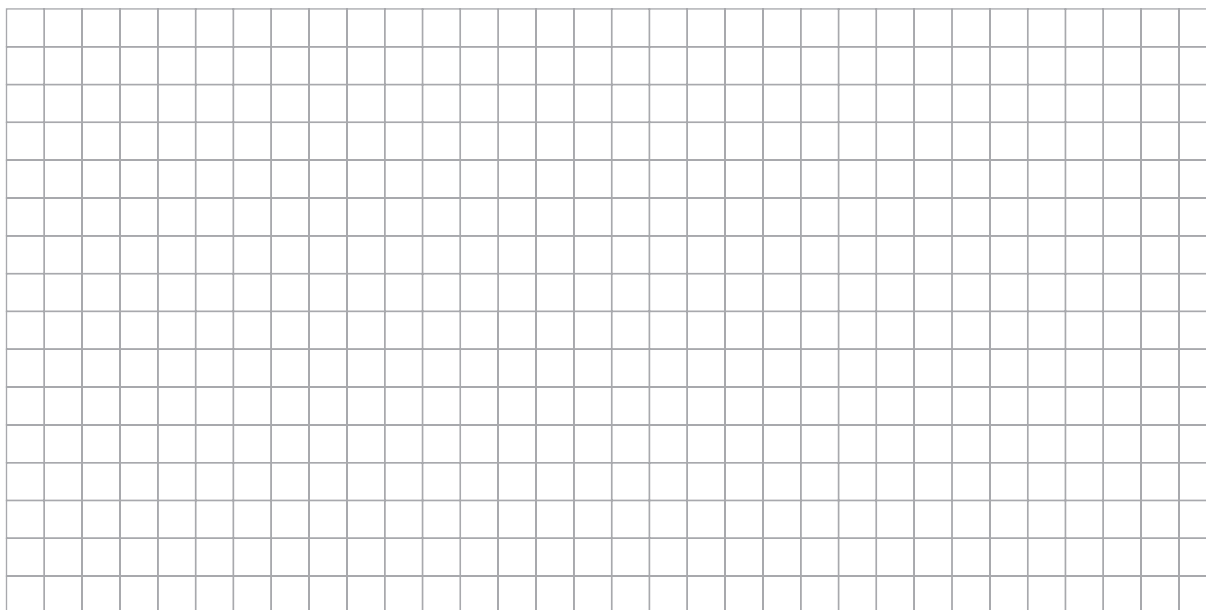
$$\text{(ii)} \quad 1^2 + 2^2 + 3^2 = \frac{3 \times 4 \times 7}{6},$$

[illegible]

(iii) $1^2 + 2^2 + 3^2 + 4^2 = \frac{4 \times 5 \times 9}{6}$.



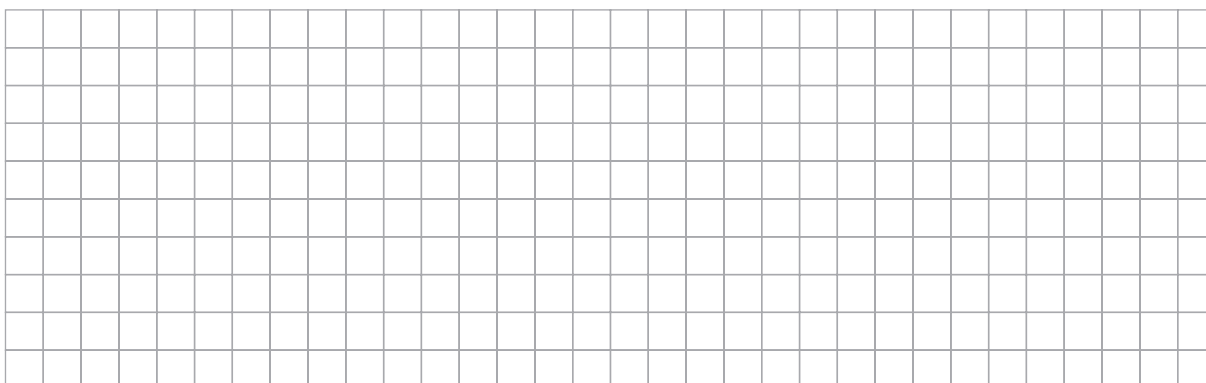
(iv) Express $1^2 + 2^2 + 3^2 + \dots + 10^2$ in the same way and hence, evaluate $1^2 + 2^2 + 3^2 + \dots + 10^2$.



(c) (i) Find a formula for $1^2 + 2^2 + 3^2 + \dots + n^2$.



(ii) Use this formula to evaluate the sum of the first 100 square numbers.



Question 2

(25 marks)

- (a) Show that the sequence with $S_n = pn^2 + qn$ is an arithmetic sequence. Find the first term and the common difference in terms of p and q .

A full-page sheet of white graph paper featuring a uniform grid of thin, light gray horizontal and vertical lines. The grid consists of small squares covering the entire area of the page.

- (b) (i) Prove that if three numbers are consecutive terms of a geometric sequence, then their logs are consecutive terms of an arithmetic sequence. Find S_n for the arithmetic sequence if a is the first term, and r is the common ratio of the geometric sequence.

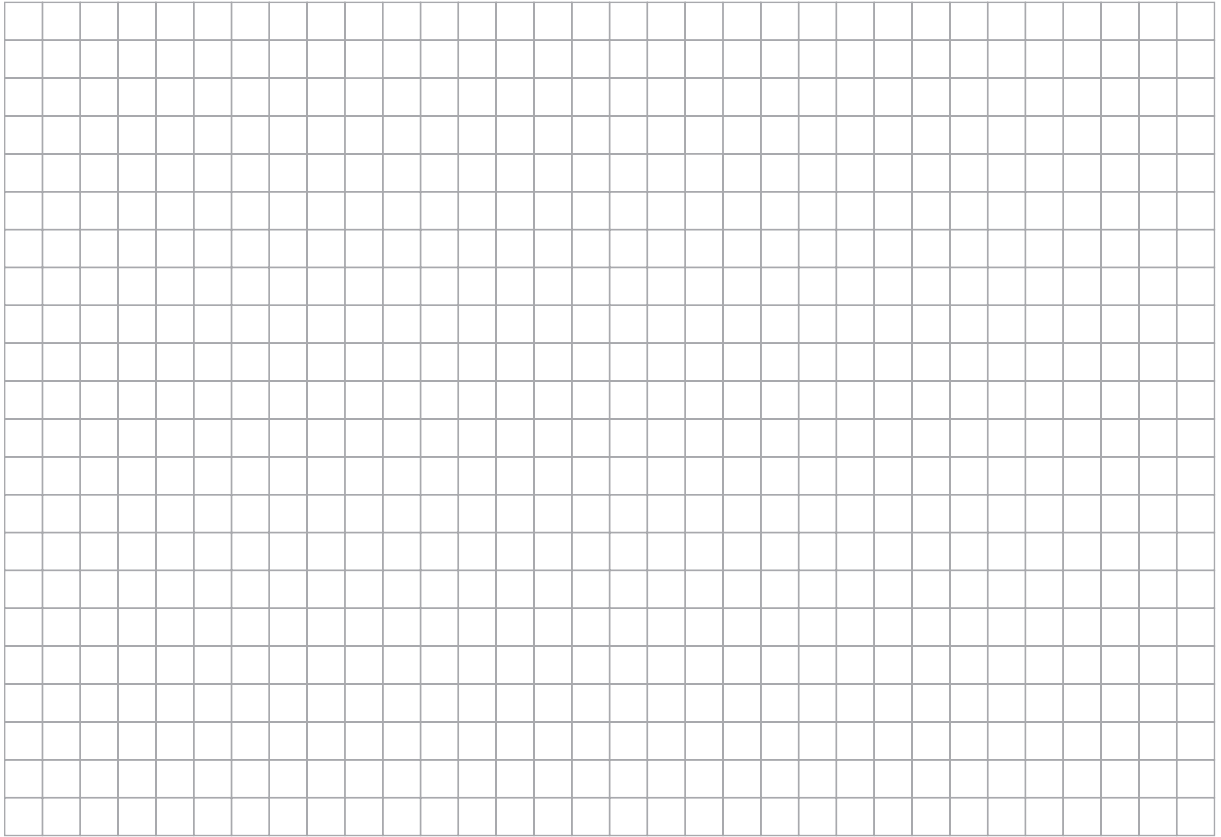
[illegible]

- (ii)** For a log to the base of 3, find r , if $a = 1$ and $S_{50} = 2450$ in **part (b) (i)**.

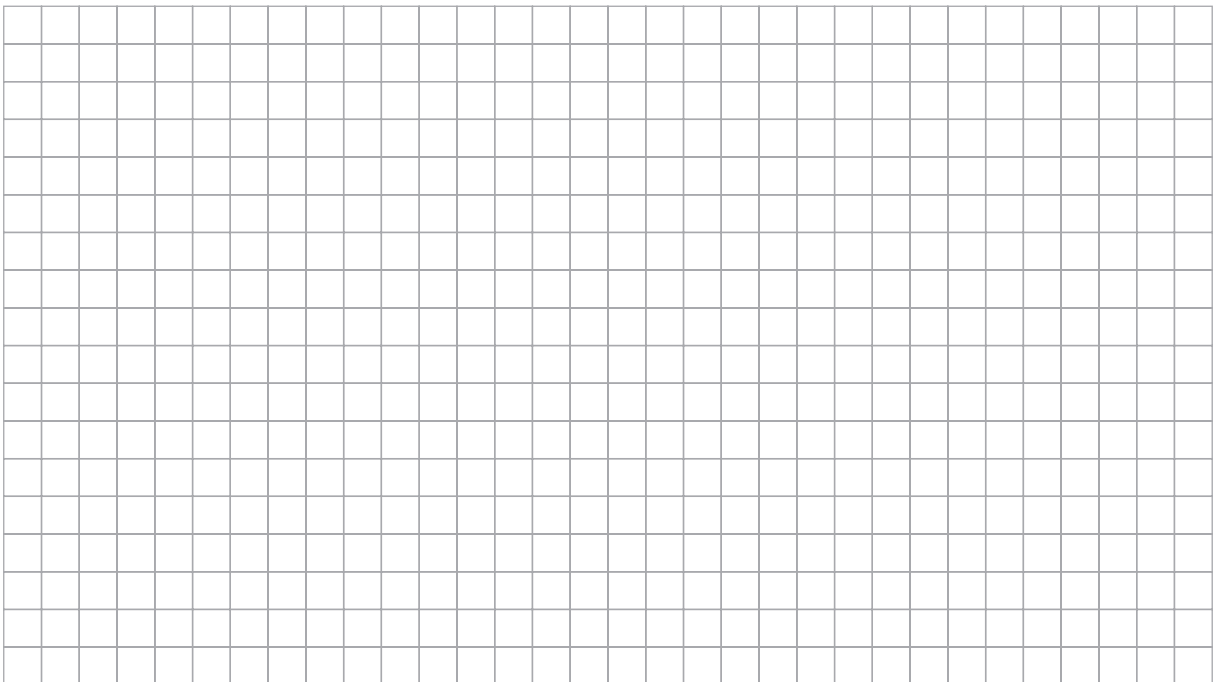
[illegible]

Question 3**(25 marks)**

- (a) Find the three solutions of $z^3 = 1$, two complex and one real, using De Moivre's theorem, or otherwise.



- (b) If 1 and ω are two of the roots in **part (a)**, show that ω^2 is the third root. Form the quadratic equation with roots ω and $\frac{1}{\omega}$.



Question 4

(25 marks)

If $x^2 - px + q$ is a factor of $x^3 + 3px^2 + 3qx + r$,

(a) show that $q = -2p^2$,

[illegible]

(b) show that $r = -8p^3$.

A large grid of graph paper with 20 columns and 10 rows. The grid is composed of small squares, with a slightly larger margin at the top for writing.

(c) Find the three roots of $x^3 + 3px^2 + 3qx + r = 0$ in terms of p .

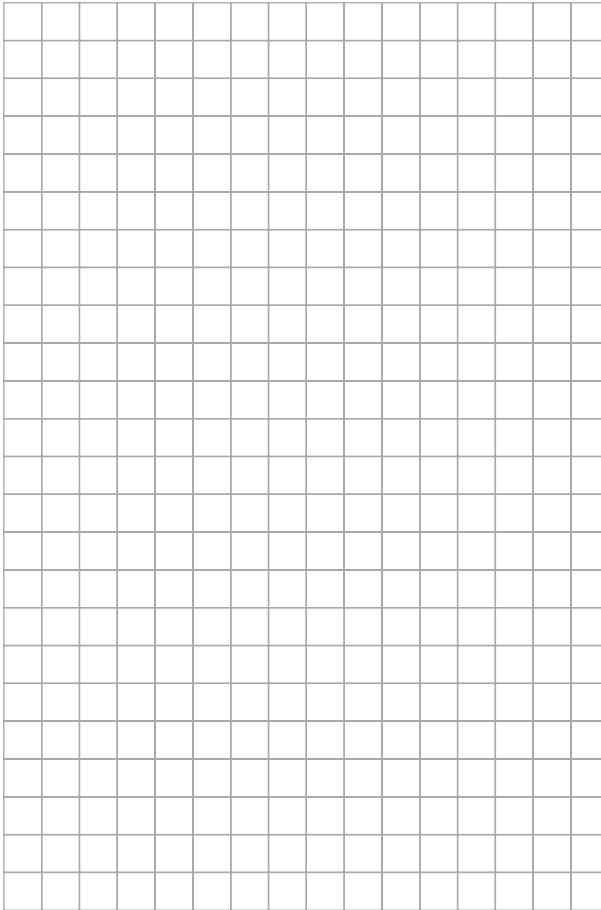
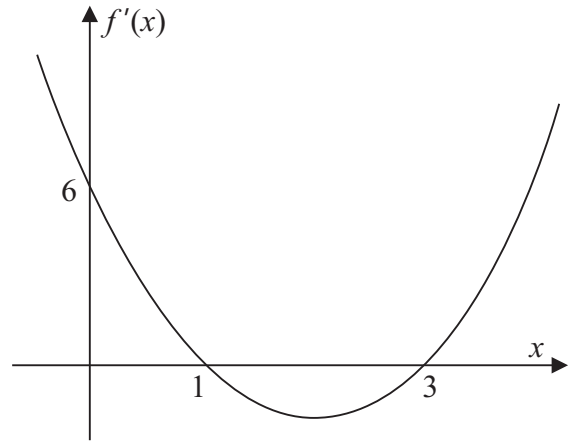
A large grid of graph paper with 20 columns and 15 rows. The grid is composed of small squares, with a slightly larger margin at the top for writing.

Question 5

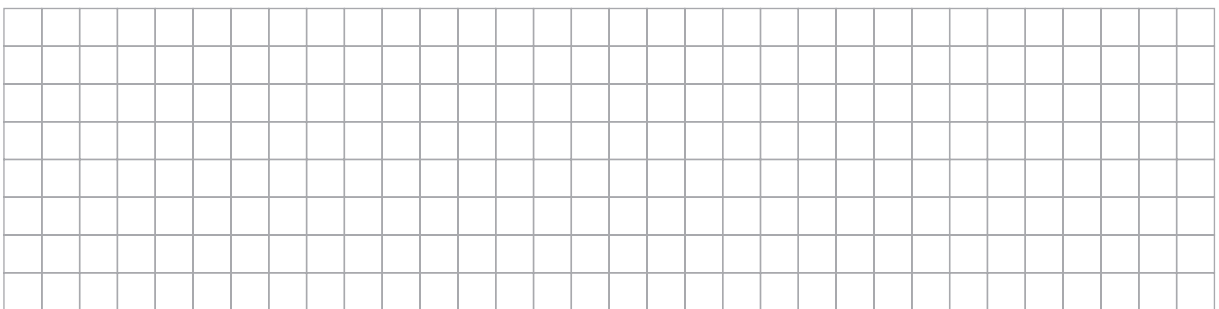
(25 marks)

The graph on the right, $f'(x) = y = ax^2 + bx + c$, is the derivative of the function $f(x)$.

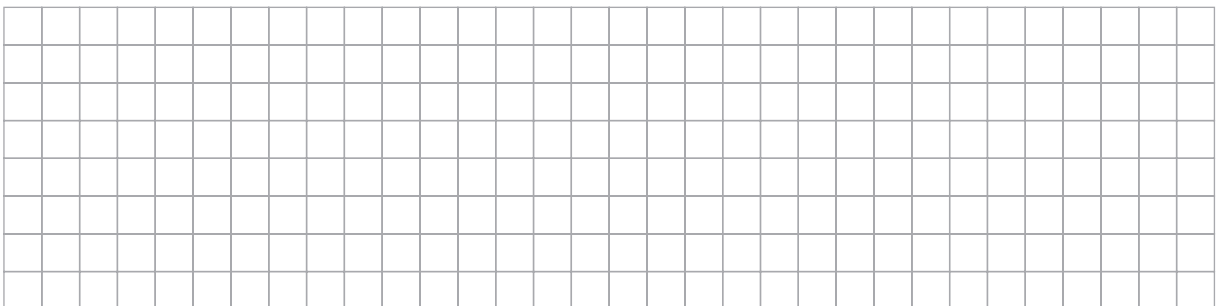
- (a) Find a , b and c .



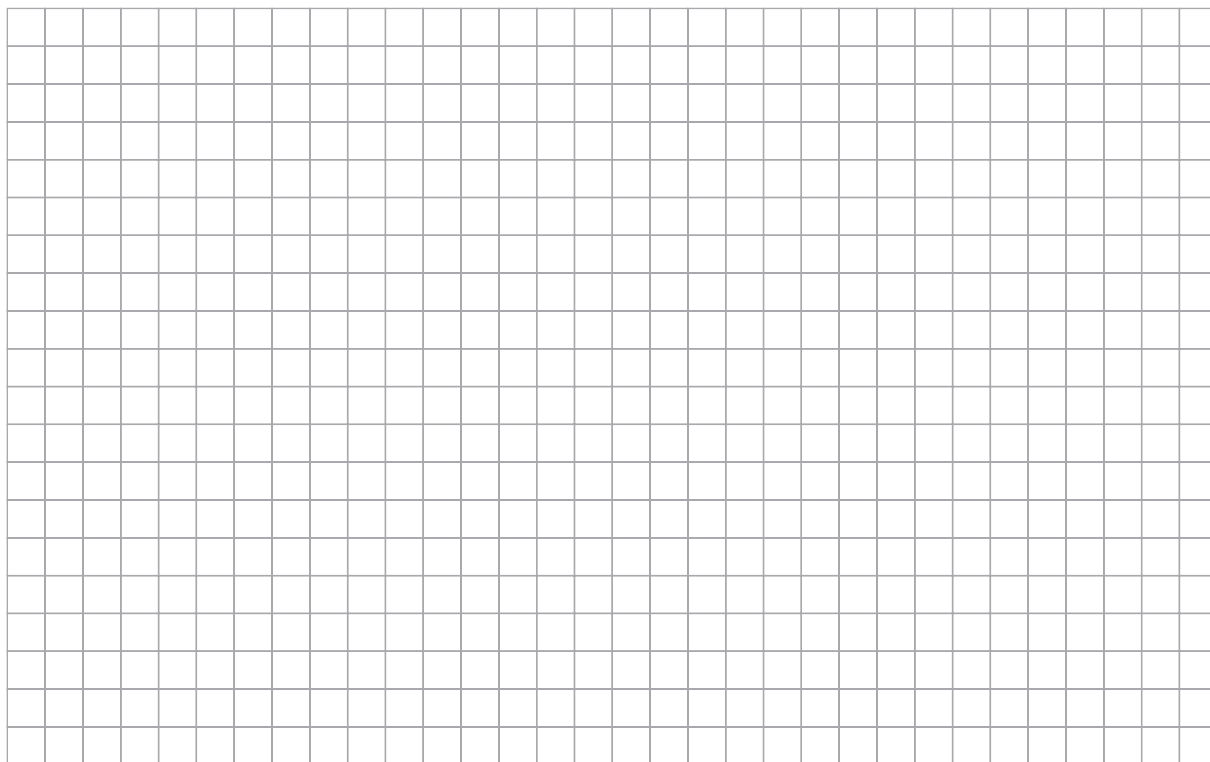
- (b) Find the x -coordinates of the local maximum and local minimum of $f(x)$ from the graph.



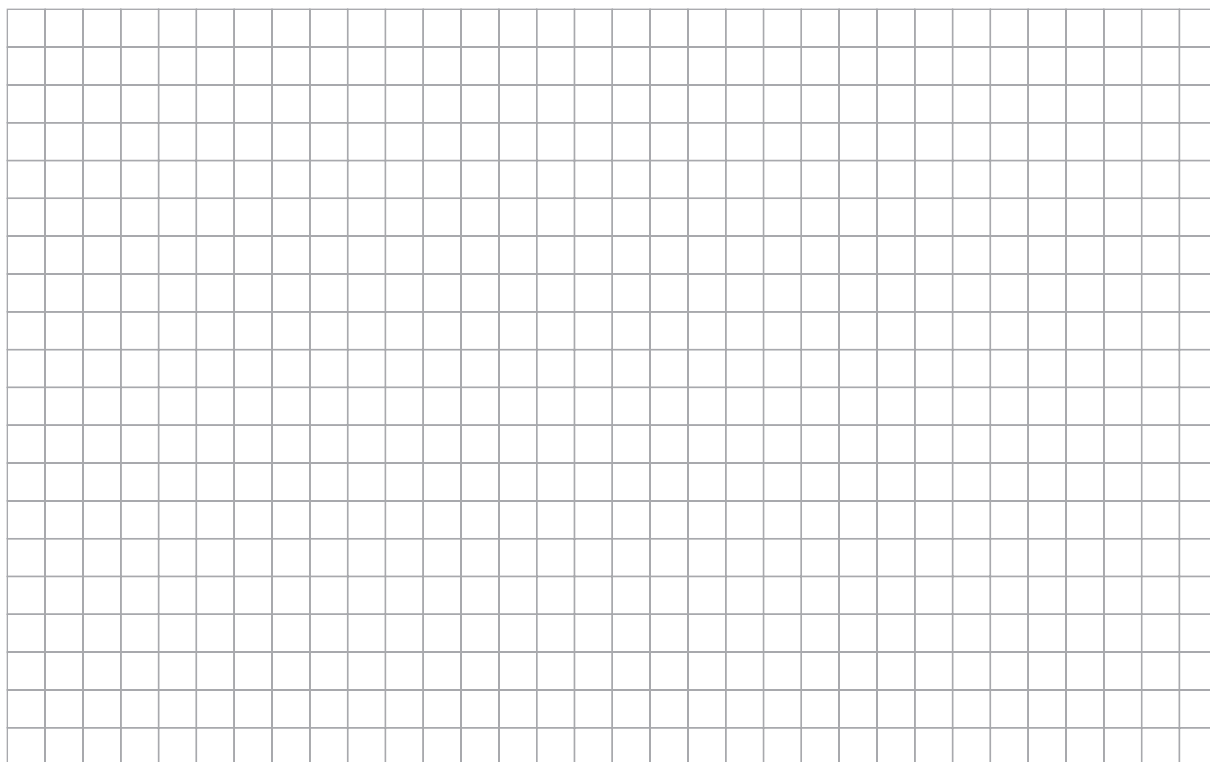
- (c) Mark the point of inflection D on the graph and calculate its x -coordinate.



- (d) Find the equation of $f(x)$ given $(0, 0)$ is on this graph.



- (e) Find the y -coordinates of the local maximum, local minimum and the point of inflection.



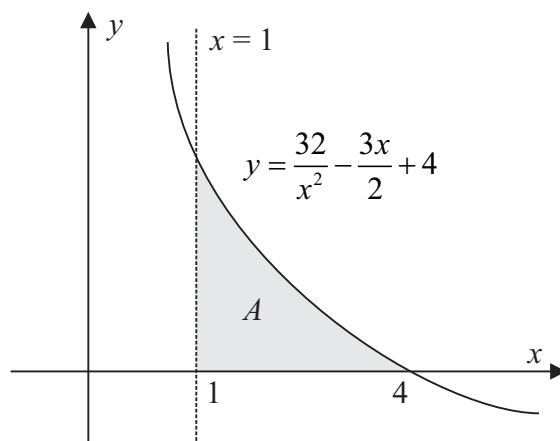
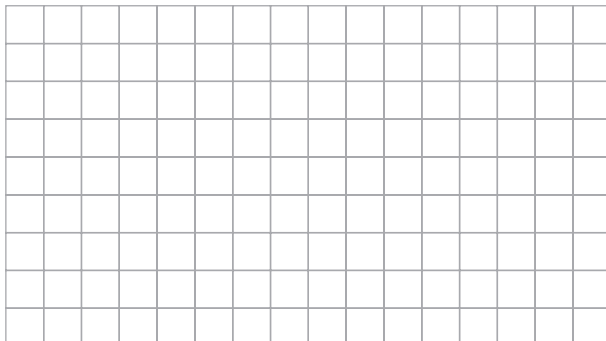
Question 6

(25 marks)

The figure shows the equation of the curve

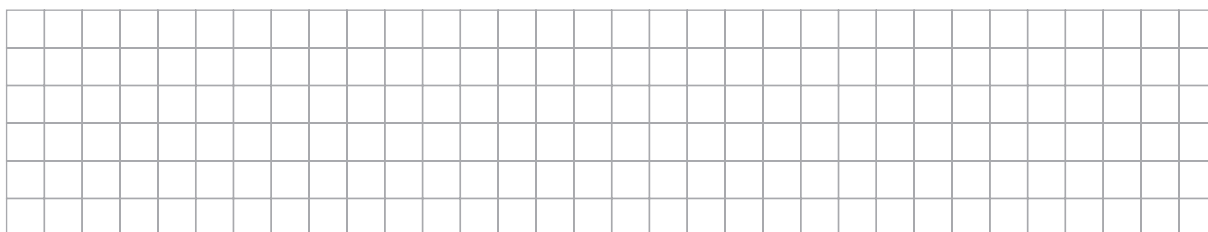
$$y = \frac{32}{x^2} - \frac{3x}{2} + 4, \quad x > 0.$$

- (a) Show the curve crosses the x -axis at $x = 4$.



- (b) Find an estimate of the area A using the trapezoidal rule. Use the table below. Calculate each value of y , to two decimal places.

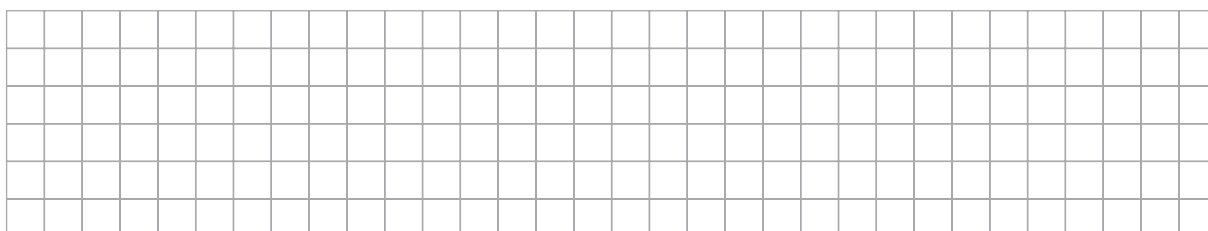
x	1	1.5	2	2.5	3	3.5	4
y							



- (c) Find the exact area by integration.



- (d) Find the percentage error that the trapezoidal estimate gives in the exact area, to the nearest percentage.



Contexts and Applications

150 marks

Answer **all three** questions from this section.

Question 7

(50 marks)

A businessman is setting up an export business. He borrows €60 000 over five years from the bank at an interest rate of 6.5% per annum. He wants to pay off the debt in equal annual payments, with the first payment one year after he takes out the loan.

- (a) Calculate his annual repayment amount using the amortisation formula on page 31 of the *Formulae and Tables* booklet.

[illegible]

- (b)** Make a payment schedule showing the annual fixed payments, the annual interest on the outstanding balance, the portion of the payment contributing towards reducing the debt, and the outstanding balance for each year.

Payment #	Fixed Payment	Interest	Debt Payment	Balance
0				€60 000
1				
2				
3				
4				
5				

[illegible]

- (c) After five years, the export business needs to raise more money to expand. They have developed a reputation as a solid company with good prospects. In order to raise money this time, the board of the company decides to issue eight-year €5000 bonds that will pay €250 at the **beginning** of every year.

The current market interest rate is 4.5% per annum.

- (i) The bond promises to do two things. The bond holder gets €5000 when the bond matures in eight years' time. What is the present value of €5000 in eight years' time?

[illegible]

- (ii)** The other thing the bond promises to do is to pay €250 per year at the beginning of each year. By writing out the present values as a sum in a geometric series, find the present value of eight payments of €250 per year for eight years.

A full-page view of a blank sheet of graph paper. The grid consists of small, uniform squares formed by thin, light gray lines. There are no margins, text, or other markings on the page.

- (iii)** Based on the previous calculation, what is the minimum price, to the nearest euro, at which the bonds can be offered?

Question 8

(50 marks)

- (a) A chemistry experiment calls for a 30% Hydrochloric (HCl) solution. This means that in a volume of 10 litres (l) of such a solution 3 l are HCl acid and 7 l are water.
- (i) If the lab has only 40% solutions and 25% solutions of HCl acid in the stores, what volume of each should be mixed to get 15 l of 30% solution?

Use the following technique to aid your solution:

Let x = Number of litres of 40% solution.

Let y = Number of litres of 25% solution.

Write down two equations in x and y and solve them.

A full-page view of a blank sheet of graph paper. The grid consists of small, uniform squares formed by thin, light gray lines. The grid covers the entire area of the page, leaving no margins or other markings.

- (ii) If another litre of water is added to the 15 l of 30% solution, what is the new concentration of the acid?

[illegible]

- (d) (i)** The pH of a solution increases by 0.5 every hour. If its initial pH value is 2, find the pH value after five hours.

[illegible]

- (ii) Show that the corresponding ion concentration decreases as a geometric sequence. Find its first term, the common ratio and when the pH value will be 8.7.

A full-page sheet of white graph paper featuring a uniform grid of thin, light gray horizontal and vertical lines. The grid consists of small squares covering the entire area of the page.

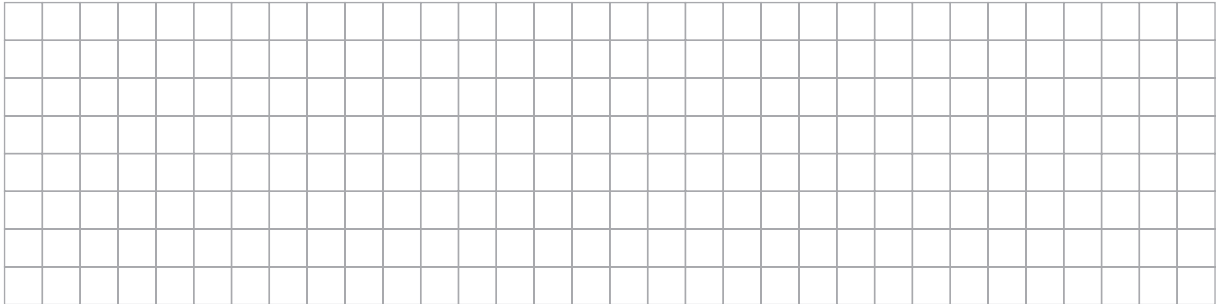
Question 9

(50 marks)

When hyperventilating, a person breathes in and out rapidly. A spirogram draws a graph of the volume V in litres in a person's lung against time t in minutes.

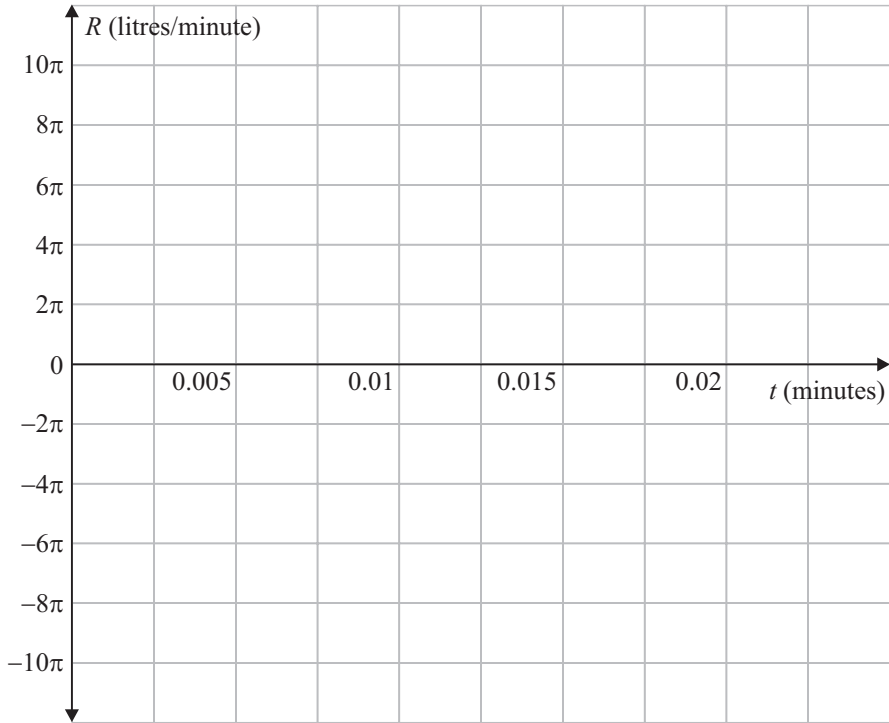
The rate R of flow of air in litres per minute is given by $R = \frac{dV}{dt} = 10\pi \sin(200\pi t)$.

(a) Find the period and range of R .



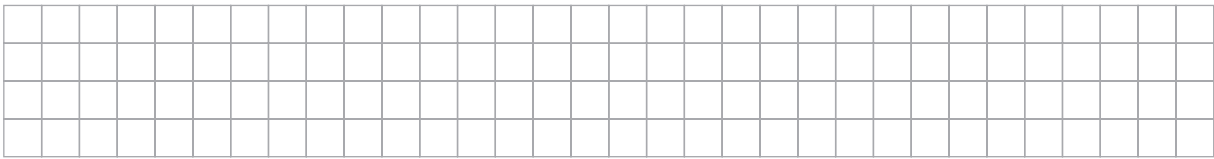
(b) Draw a graph of R against t for $0 \leq t \leq 0.02$.

t	0	0.0025	0.005	0.0075	0.01	0.0125	0.015	0.0175	0.02
$200\pi t$									
$\sin(200\pi t)$									
$10\pi \sin(200\pi t)$									

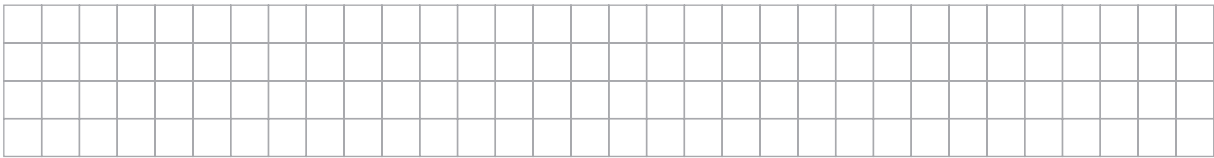


(c) State from the graph:

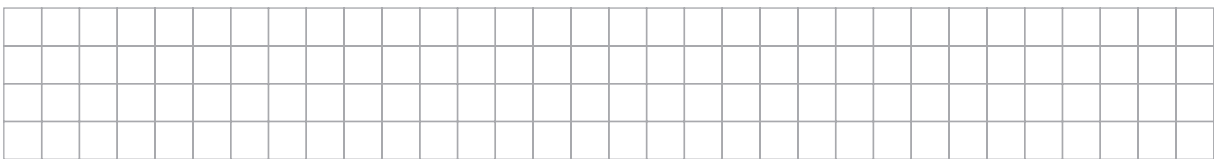
(i) the time intervals when the person is breathing in,



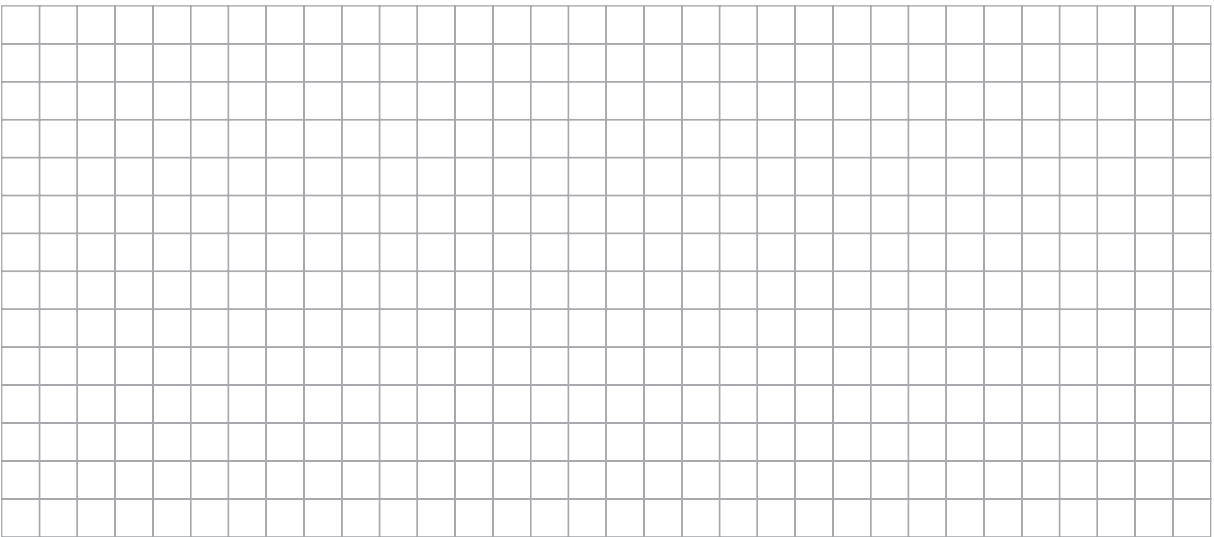
(ii) the time intervals when the person is breathing out,



(iii) the times when the rate of flow is a maximum value when a person is breathing in. Mark these maximum points M on the graph.



(d) Write a formula for volume V in terms of time t if at $t = 0$, $V = 2.95$ litres.



(e) Calculate the average volume of air in the lungs over a cycle of breathing in and out.

